3.5 - Solving Linear Systems with THREE variables

Learning Targets for today

- To be able to solve linear systems with three variables (x, y, and z) using substitution.
- To be able to solve linear systems with three variables (x, y, and z) using elimination.

Vocabulary

Linear System (3 equations) – three equations that all contain "x, y, and z".

Ex:

$4x+2y+3z=1$

$2x-3y+5z=-14$

$6x-y+4z=-1$

Solution of the system – An order pair (x, y, z) that makes all equations in the system true!

Ex: $(2, 1, -3)$

x y z

Three Different Types of Solutions – One Solution (x, y, z), Infinitely Many solutions, or No Solutions.

- If you obtain a false equation such as $0 = 1$ at any time in your problem, then the system has NO SOLUTIONS.
- If you obtain a statement such as $1 = 1$ that is an identity statement, and that tells you that your system has MANY SOLUTIONS.

Solving by Elimination

1. Work to eliminate one of the variables using two equations. Then use two different equations to eliminate that same variable.
2. Once you do this you will have two NEW equations that you can then work with.

Solve by Elimination

Solve the system by using the elimination method.

1. $3x - 2y + 4z = 20$
   $-x + 5y + 12z = 73$
   $x + 3y - 2z = 1$

$3(-x + 5y + 12z = 73)$

$3x - 2y + 4z = 20$

$-3x + 15y + 36z = 219$

$8y + 10z = 74$ *

$13y + 40z = 239$ *

$(2, 3, 5)$

$-4(8y + 10z = 74)$

$13y + 40z = 239$

$-32y - 40z = -296$

$13y + 40z = 239$

$-19y = -57$

$\frac{-19y}{-19} = \frac{-57}{-19}$

$y = 3$ ✓

$8(3) + 10z = 74$

$24 + 10z = 74$

$-24 \quad -24$

$10z = 50$

$\frac{10z}{10} = \frac{50}{10}$

$z = 5$ ✓

$x + 3y - 2z = 1$

$x + 3(3) - 2(5) = 1$

$x + 9 - 10 = 1$

$x - 1 = 1$

$+1 \quad +1$

$x = 2$ ✓

Solve by Elimination

Solve the system by using the elimination method.

1. $4x + 2y + 3z = 1$
   $6x - y + 4z = -1$
   $2x - 3y + 5z = -14$

Solve by Substitution

Solve the system by using the substitution method.

1. $2x + y + z = 8$
$-x + 3y - 2z = 3$
$y = x + z$

$2x + (x+2) + (2) = 8$
$-x + 3(x+2) - 2z = 3$
$3x + 32$

$(-2, 5, 7)$

$3x + 2z = 8$
$-2(2x + z = 3)$

$3x + 2z = 8$
$-4x - 2z = -6$

$-x = 2$
$\frac{-x}{-1} = \frac{2}{-1}$
$x = -2$ ✓

$2(-2) + z = 3$
$-4 + z = 3$
$+4 \quad +4$
$z = 7$ ✓

$y = x + z$
$y = -2 + 7$
$y = 5$

Solve the Linear System with Three Variables – YOUR TURN!!

Solve the system by using the elimination method.

1. $4x - 3y + 5z = -19$
   $3x - y - 8z = -21$
   $-2x + y + 3z = 13$

$4x - 3y + 5z = -19$

$3(-2x + y + 3z = 13)$

$-6x + 3y + 9z = 39$

$X - 5z = -8$ *

$-2x + 14z = 20$ *

$2(X - 5z = -8)$

$-2x + 14z = 20$

$2x - 10z = -16$

$-2x + 14z = 20$

$4z = 4$

$\frac{4z}{4} = \frac{4}{4}$

$z = 1$ ✓

$X - 5(1) = -8$

$X - 5 = -8$

$+5 \quad +5$

$X = -3$ ✓

$-2x + y + 3z = 13$

$-2(-3) + y + 3(1) = 13$

$6 + y + 3 = 13$

$y + 9 = 13$

$-9 \quad -9$

$y = 4$ ✓

$(-3, 4, 1)$

Solve the Linear System with Three Variables – YOUR TURN!!

Solve the system by using the substitution method.

1. $5x + 3y + z = 56$
   $x + y + z = 24$
   $z = x + y$

$5x + 3y + (x+y) = 56$

$x + y + (x+y) = 24$

$(4, 8, 12)$

$6x + 4y = 56$

$-2(2x + 2y = 24)$

$6x + 4y = 56$

$-4x - 4y = -48$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$ ✓

$2(4) + 2y = 24$

$8 + 2y = 24$

$-8 \quad -8$

$\frac{2y}{2} = \frac{16}{2}$

$y = 8$ ✓

$z = x + y$

$z = 4 + 8$

$z = 12$

3.4	Linear Programming	❑ To be able to solve a problem using linear programming.	
3.5	Systems with Three Variables	❑ To be able to solve systems with three variable using the elimination method. ❑ To be able to solve systems with three variable using the substitution method.	Pg. 171 #10 - #14 (EVEN)
3.6	Solving Systems Using Matrices	❑ To be able to represent a system of linear equations with matrices. ❑ To be able to solve a system of linear equations with matrices.	

***Assignments could change day by day. If you are absent, assume the next assignment is due. Additional worksheets or problems could be given at any time.