

10.4 Ellipses

Learning Targets for today ✖

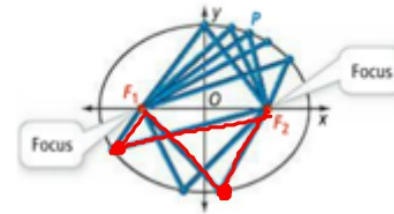
- ① To be able to write an equation of an ellipse.
- ① To be able to find the foci of an ellipse.
- ① To be able to graph an ellipse.
- ① To be able to use the foci of an ellipse.

"C"

Key Concepts!

Ellipse – a set of all points P in a plane such that the sum of the distance from P to two fixed points is a constant (k)

Focus (Foci) - the point or set of fixed points



Major Axis – Contains the foci and has its endpoints

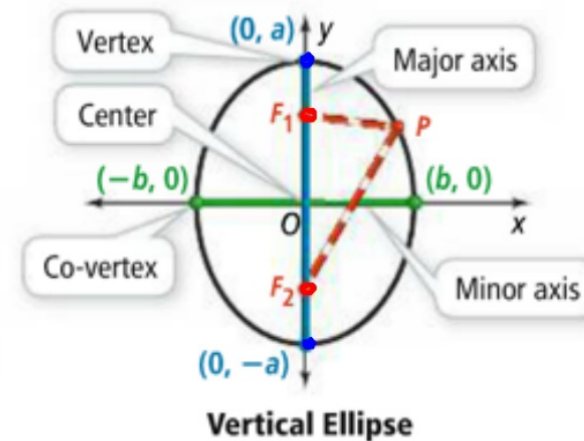
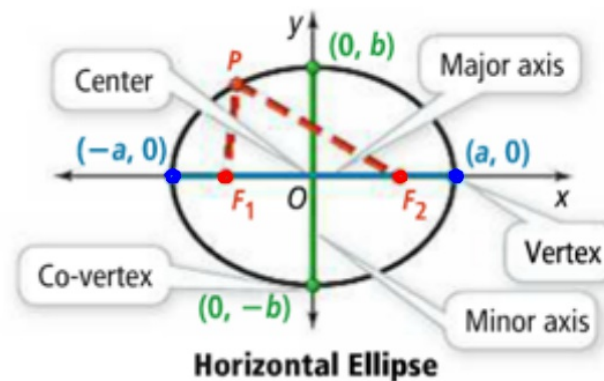
Vertices – the endpoints of the major axis

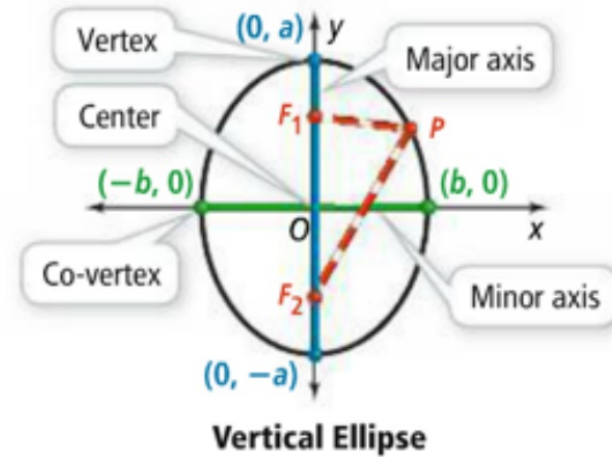
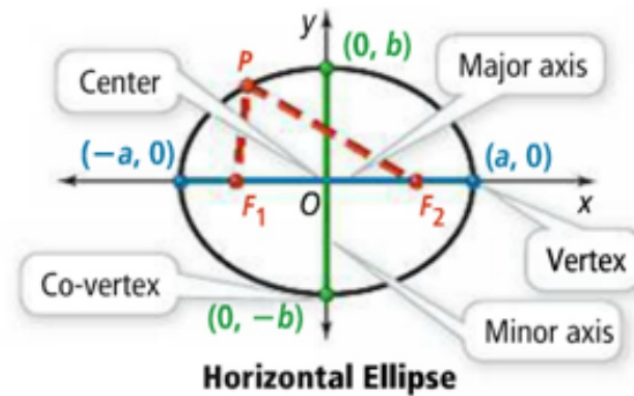
" a "

Minor Axis – is perpendicular to the major axis at the center of the ellipse.

Co-Vertices – the endpoints of the minor axis

" b "





	Horizontal Ellipses	Vertical Ellipses
Standard Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a > b > 0$
Major Axis	horizontal	vertical
Vertices "a"	$(\pm a, 0)$ $(a, 0)$ $(-a, 0)$	$(0, \pm a)$
Co-vertices "b"	$(0, \pm b)$ $(0, b)$ $(0, -b)$	$(\pm b, 0)$
Foci "c"	$(\pm c, 0)$ on x-axis $(c, 0)$ $(-c, 0)$	$(0, \pm c)$ on y-axis

$$c^2 = a^2 - b^2$$

Writing an Equation of an Ellipse

Example for you...

1. What is an equation in standard form of an ellipse centered at the origin with vertex $(-5, 0)$ and co-vertex $(0, 2)$?

$$(5, 0)$$

$$a = 5$$

$$a^2 = 25 \checkmark$$

$$(0, -2)$$

$$b = 2$$

$$b^2 = 4 \checkmark$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

Your turn to try...

1. What is an equation in standard form of an ellipse centered at the origin with vertex $(0, 6)$ and co-vertex $(-3, 0)$?

$$(0, 6) \text{ (y-axis)}$$

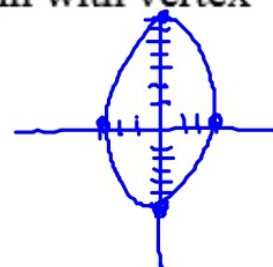
$$a = 6$$

$$a^2 = 36$$

$$(-3, 0)$$

$$b = 3$$

$$b^2 = 9$$



$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

Finding the Foci of an Ellipse

Example for you...

What are the foci of the ellipse with the given equation? Graph both foci and ellipse.

$$1. \frac{36x^2}{3600} + \frac{100y^2}{3600} = \frac{3600}{3600}$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

$$a^2 = 100 \quad b^2 = 36$$

$$a = 10 \quad b = 6$$

$$\boxed{(10, 0)} \\ \boxed{(-10, 0)}$$

$$\boxed{(0, 6)} \\ \boxed{(0, -6)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

foci

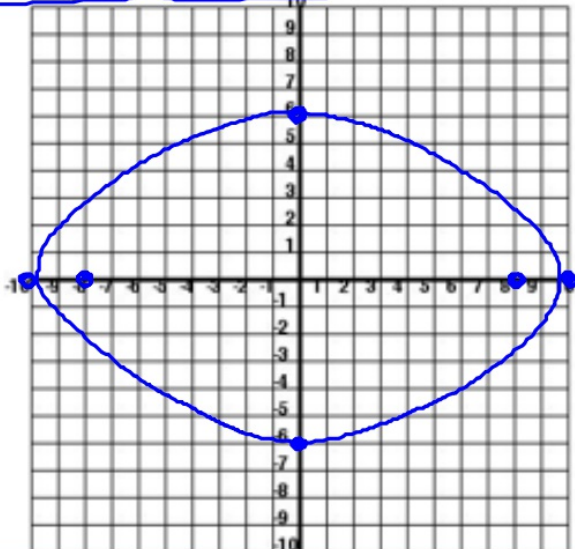
$$c^2 = a^2 - b^2$$

$$c^2 = 100 - 36$$

$$\sqrt{c^2} = \sqrt{64}$$

$$c = 8$$

$$\boxed{(8, 0)} \quad \boxed{(-8, 0)}$$



Your turn to try...

What are the foci of the ellipse with the given equation? Graph both foci and ellipse.

$$1. \frac{25x^2}{225} + \frac{9y^2}{225} = \frac{225}{225}$$

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

$$b^2 = 9 \quad a^2 = 25$$

$$b = 3 \quad a = 5$$

$$\boxed{(3, 0)} \\ \boxed{(-3, 0)}$$

$$\boxed{(0, 5)} \\ \boxed{(0, -5)}$$

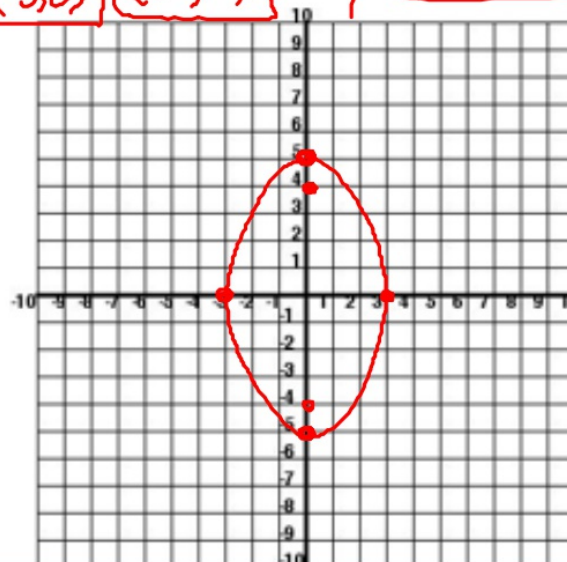
$$\text{foci} \\ c^2 = a^2 - b^2$$

$$c^2 = 25 - 9$$

$$\sqrt{c^2} = \sqrt{16}$$

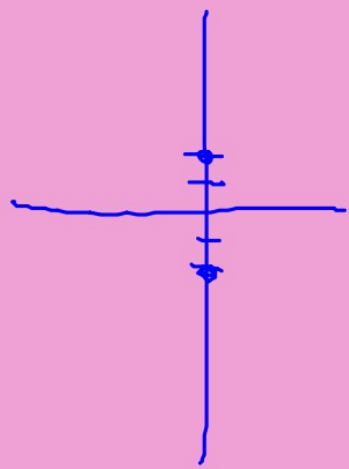
$$c = 4$$

$$\boxed{(0, 4)} \\ \boxed{(0, -4)}$$



Using the Foci of an Ellipse

Example for you...	Your turn to try...
<p>Write the standard equation of an ellipse with the foci of $(\pm 5, 0)$ and co-vertex of $(0, \pm 2)$ (<i>y-axis</i>)</p> <p>$c = 5$ $c^2 = 25$</p> <p>$b = 2$ $b^2 = 4$</p> <p>$a^2 = ?$ $c^2 = a^2 - b^2$ $25 = a^2 - 4$ $\begin{array}{r} +4 \\ \hline 29 = a^2 \end{array}$</p> <p>$\frac{x^2}{29} + \frac{y^2}{4} = 1$</p>	<p>Write the standard equation of an ellipse with the foci of $(0, \pm 3)$ and co-vertex of $(\pm 1, 0)$ (<i>x-axis</i>)</p> <p>$c = 3$ $c^2 = 9$</p> <p>$b = 1$ $b^2 = 1$</p> <p>$a^2 = ?$ $c^2 = a^2 - b^2$ $9 = a^2 - 1$ $\begin{array}{r} +1 \\ \hline 10 = a^2 \end{array}$</p> <p>$\frac{x^2}{1} + \frac{y^2}{10} = 1$</p>

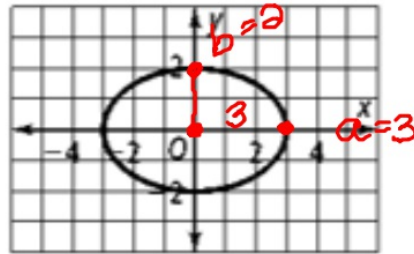


Writing Equations of Ellipses

Example for you...

Write the equation of the ellipse shown in the graph.

1.



$$a=3 \quad b=2$$
$$a^2=9 \quad b^2=4$$

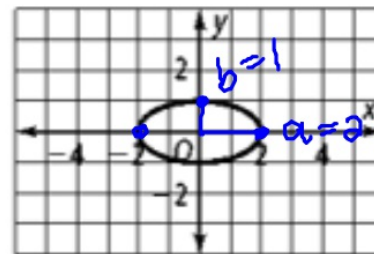
$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Your turn to try...

Write the equation of the ellipse shown in the graph.

1.



$$a=2 \quad b=1$$
$$a^2=4 \quad b^2=1$$

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Write the equation of the ellipse with the given characteristic.

2. height 12 in., width 4 in.

Write the equation of the ellipse with the given characteristic.

2. height 16 in., width 18in.