

10.5 Hyperbolas

Learning Targets for today

- ① To be able to write an equation of an ellipse.
- ① To be able to find the foci of an ellipse.
- ① To be able to graph an ellipse.
- ① To be able to use the foci of an ellipse.

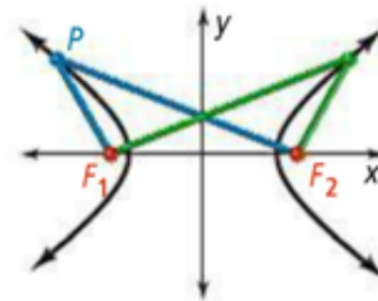
Key Concepts!

Hyperbola – the set of points P in a plane such that the absolute value of the difference between the distance from P to two fixed points F_1 and F_2 is a constant (k).

$$|PF_1 - PF_2| = k$$

Focus (Foci) – Each fixed point

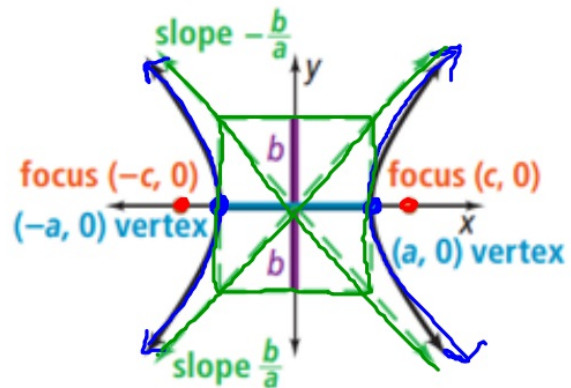
* *A Hyperbola consists of two smooth branches*



Vertex – The turning point of each branch of the hyperbola

Transverse Axis – the segment containing the two vertices

Horizontal Hyperbola



Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

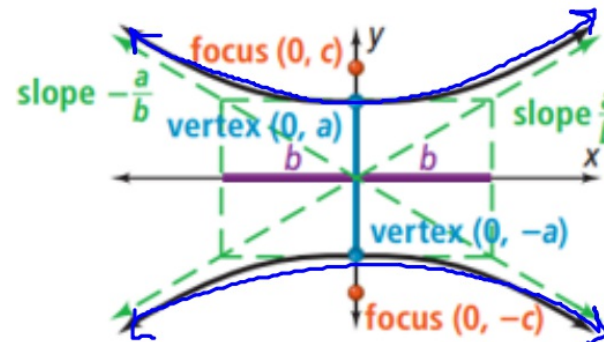
Transverse axis: Horizontal

Vertices $(\pm a, 0)$

Foci: $(\pm c, 0)$, where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{b}{a}x$

Vertical Hyperbola



Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Transverse axis: Vertical

Vertices $(0, \pm a)$

Foci: $(0, \pm c)$, where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{a}{b}x$

Graphing Hyperbolas

Example for you...

Graph the following hyperbolas.

Horizontal

$$1. \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$a^2 = 16 \quad b^2 = 9$$

$$a = 4 \quad b = 3$$

$$(4, 0) \quad (0, 3)$$

$$(-4, 0) \quad (0, -3)$$

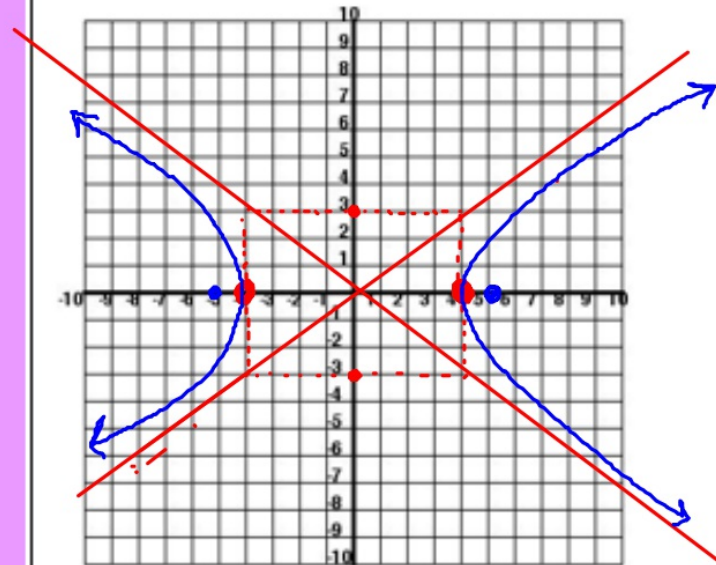
foci?
 $c^2 = a^2 + b^2$

$$c^2 = 16 + 9$$

$$\sqrt{c^2} = \sqrt{25}$$

$$c = 5$$

$$(5, 0) \quad (-5, 0)$$



Your turn to try...

Graph the following hyperbolas.

$$1. \frac{x^2}{25} - \frac{y^2}{36} = 1$$

$$a^2 = 25 \quad b^2 = 36$$

$$a = 5 \quad b = 6$$

$$(5, 0) \quad (0, 6)$$

$$(-5, 0) \quad (0, -6)$$

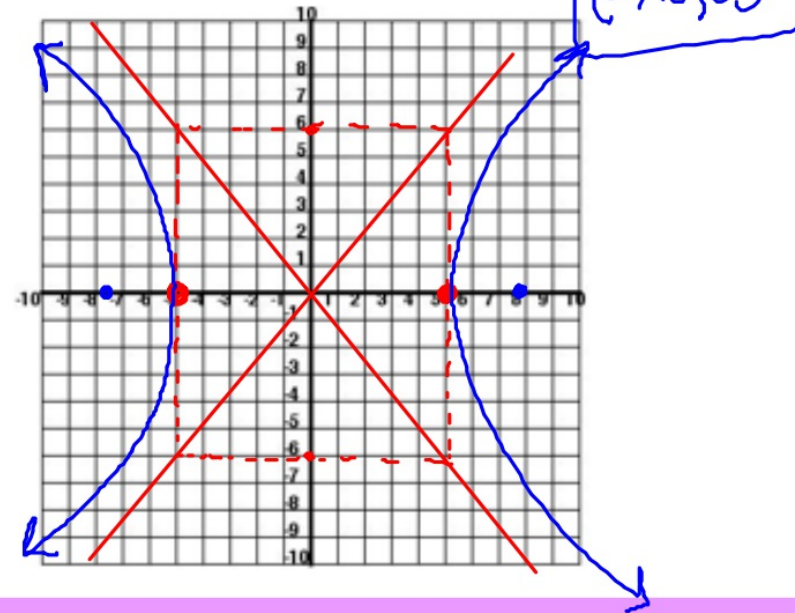
foci?
 $c^2 = a^2 + b^2$

$$c^2 = 25 + 36$$

$$\sqrt{c^2} = \sqrt{61}$$

$$c = 7.8$$

$$(7.8, 0) \quad (-7.8, 0)$$



vertical

$$2. \frac{y^2}{4} - \frac{x^2}{49} = 1$$

$$a^2=4$$

$$a=2$$

$$(0, 2)$$

$$(0, -2)$$

$$b^2=49$$

$$b=7$$

$$(7, 0)$$

$$(-7, 0)$$

foci?

$$c^2 = a^2 + b^2$$

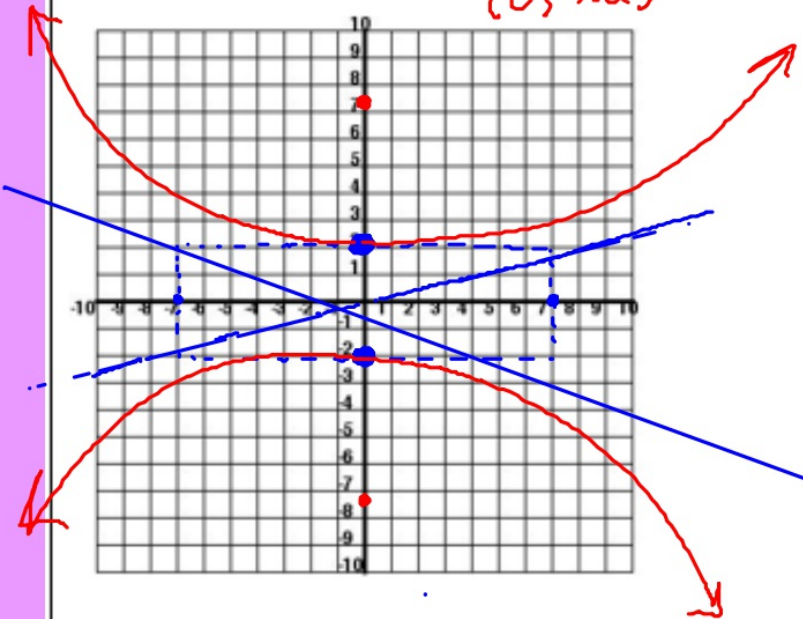
$$c^2 = 4 + 49$$

$$\sqrt{c^2} = \sqrt{53}$$

$$c = 7.2$$

$$(0, 7.2)$$

$$(0, -7.2)$$



$$2. \frac{y^2}{1} - \frac{x^2}{25} = 1$$

$$a^2=1$$

$$a=1$$

$$(0, 1)$$

$$(0, -1)$$

$$b^2=25$$

$$b=5$$

$$(5, 0)$$

$$(-5, 0)$$

foci?

$$c^2 = a^2 + b^2$$

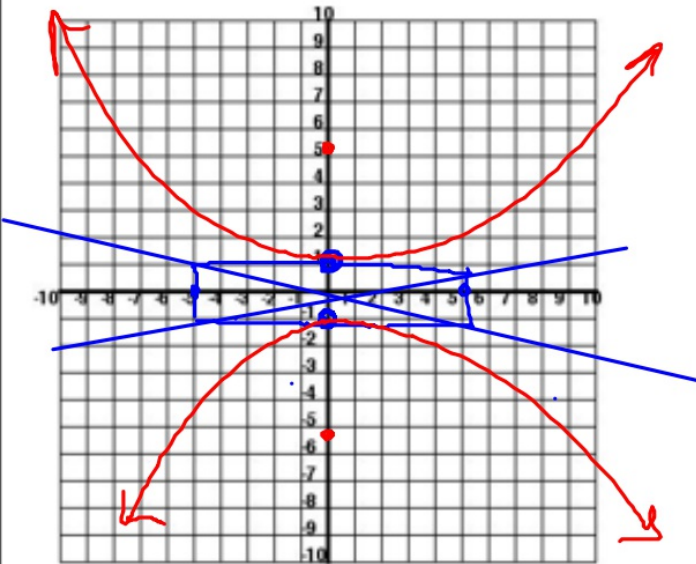
$$c^2 = 1 + 25$$

$$\sqrt{c^2} = \sqrt{26}$$

$$c = 5.1$$

$$(0, 5.1)$$

$$(0, -5.1)$$



Writing Equations of Hyperbolas

Example for you...

1. Write an equation for a hyperbola centered at $(0,0)$ that has the vertex of $(4,0)$ and $(-4,0)$ and one focus of $(5,0)$.

$$a = 4 \uparrow$$

$$a^2 = 16$$

(x-axis)

$$c = 5$$

$$c^2 = 25$$

$$b^2 = ?$$

$$c^2 = a^2 + b^2$$

$$25 = 16 + b^2$$

$$\begin{array}{r} -16 \quad -16 \\ \hline 9 = b^2 \end{array}$$

$$\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$$

Your turn to try...

1. Write an equation for a hyperbola centered at $(0,0)$ that has the vertex of $(5,0)$ and $(-5,0)$ and one focus of $(6,0)$.

$$a = 5 \uparrow$$

$$a^2 = 25$$

(x-axis)

$$c = 6$$

$$c^2 = 36$$

$$b = ?$$

$$c^2 = a^2 + b^2$$

$$36 = 25 + b^2$$

$$\begin{array}{r} -25 \quad -25 \\ \hline 11 = b^2 \end{array}$$

$$\boxed{\frac{x^2}{25} - \frac{y^2}{11} = 1}$$

2. Write an equation for a hyperbola centered at (0,0) that has the vertex of (0,-1) and (0,1) and one focus of (0,2).

$$\begin{aligned} a &= 1 \\ a^2 &= 1 \\ &\text{(y-axis)} \end{aligned}$$

$$\begin{aligned} c &= 2 \\ c^2 &= 4 \\ b^2 &? \end{aligned}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 4 &= 1 + b^2 \\ \hline 3 &= b^2 \end{aligned}$$

$$\boxed{\frac{y^2}{1} - \frac{x^2}{3} = 1}$$

2. Write an equation for a hyperbola centered at (0,0) that has the vertex of (0,4) and (0,-4) and one focus of (0,6).

$$\begin{aligned} a &= 4 \\ a^2 &= 16 \\ &\text{(y-axis)} \end{aligned}$$

$$\begin{aligned} c &= 6 \\ c^2 &= 36 \\ b^2 &? \end{aligned}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 36 &= 16 + b^2 \\ \hline 20 &= b^2 \end{aligned}$$

$$\boxed{\frac{y^2}{16} - \frac{x^2}{20} = 1}$$